Supplementary Material for Fast Multi-label Feature Selection via Global Relevance and Redundancy Optimization

I. Parameter Analysis

In this section, we study the influence of parameters involved in GRROfast and GRRO. For each of them, we vary one parameter while the others are fixed at their best setting.

In GRROfast, there are four parameters to optimize, including regularization parameter α_2 for balancing the feature redundancy term, the percentage $p\%$ of promising labels in original label set, the percentage $q\%$ of candidate relevant features in original feature set, and the number k of label-specific features. Fig. 1 shows the experimental result on all the data sets in terms of α_2, p, and q, in which the X-axis indicates the value of parameters, and the Y-axis indicates the average classification result (ACR), which can be estimated by Eq. (17). Note that $p1$, $p2$,..., and $p7$ correspond to the values of α_2, $p\%$, and $q\%$ respectively. Specially, $p1$, $p2$,..., and $p7$ equal to 10^{-3}, 10^{-2},..., and 10^3 respectively while we analyze the influence of α_2, whereas they equal to 10%, 25%,..., and 100% respectively while we analyze the influence of p and q. From Fig. 1, we can see: (1) When α_2 becomes larger the performance improves. α_2 tends to be set as the value more than 10^1; (2) While varying p or q, generally the ACR is going to change dramatically, hence GRROfast is sensitive to the two parameters. According to Fig. 1, we further give the reduction $w.r.t.$ labels and features on all the data sets, as shown in TABLE I. From TABLE I, we observe that the number of labels can be reduced on most of data sets to obtain the best performance, i.e., $p\% < 100\%$. In terms of q, a larger percentage $q\%$ contributes to a good performance on a few data sets, such as *Coffee*, *Corel5k*, and *Langlog*, but on the other data sets, q tends to be smaller, e.g., $q\% = 10\%$. In a nutshell, GRROfast can avoid many ineffective calculations in terms of labels and features. Besides, TABLE II lists the distribution of optimal k on all the data sets. We can conclude from TABLE II that generally a small value of k, e.g., $k = 5$, helps to the label-specific feature selection.

GRRO also includes parameter k. In addition to k, two regularization parameters α_1 and β are used to reflect the influence of feature redundancy and label correlation respectively. The optimal value distribution of k is similar with that in GRROfast, so we only analyze α_1 and β on all the data sets. For brevity, the experimental results are reported on the Web[1]. Based on the results, we can see that α_1 and α_2 (involved in GRROfast) have the similar impact to the performance, this is because that both α_1 and α_2 are used to control the influence of feature redundancy. Besides, β tends to be smaller than α_1 for obtaining a good performance.

[1]https://jiazhang-ml.pub/Supplement4-GRROfast.pdf

TABLE I
The Reduction in Terms of Labels and Features on All the Data Sets

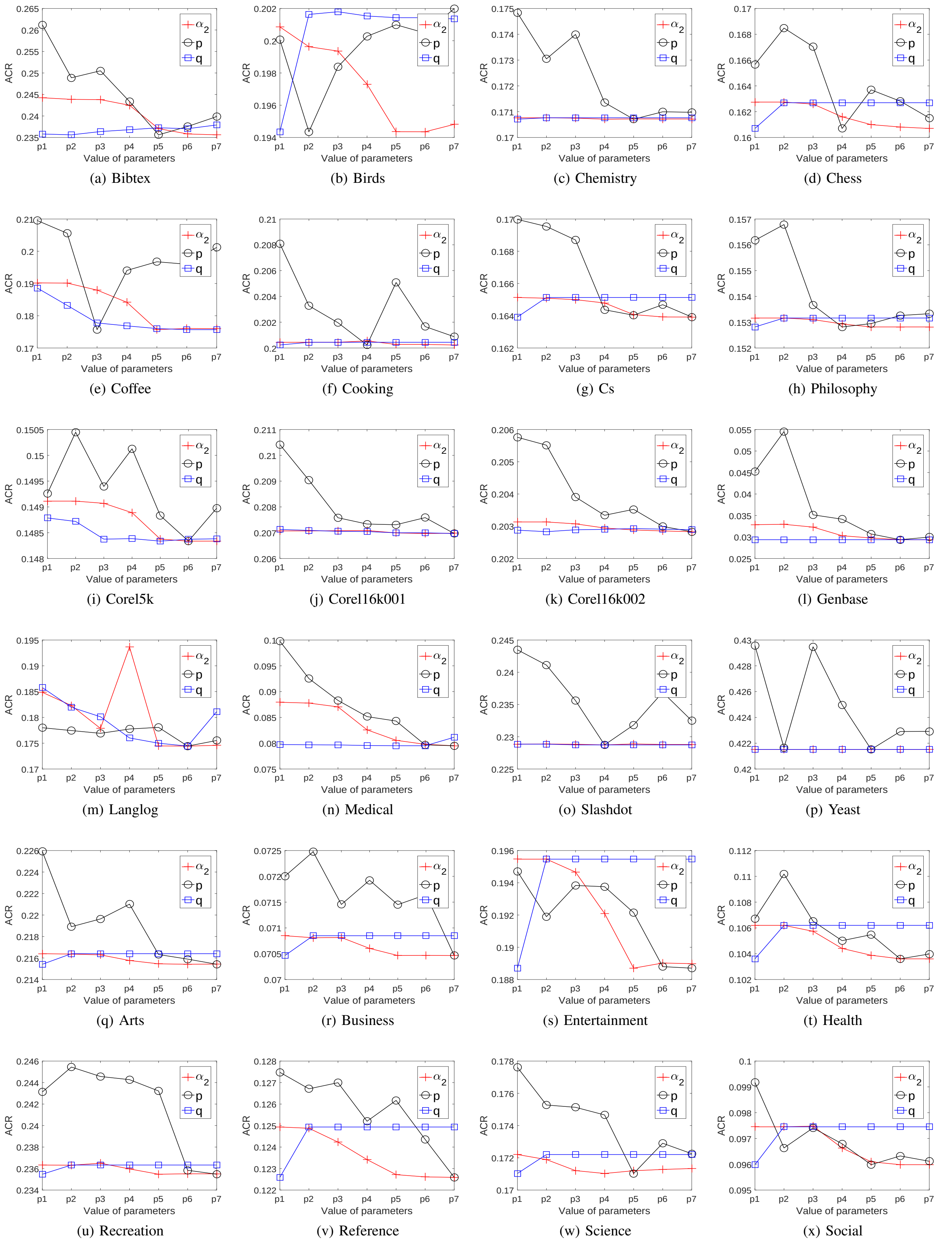
Data set	The reduction of labels (1-p%)	The reduction of features (1-q%)
Bibtex	30%	75%
Birds	75%	90%
Chemistry	30%	90%
Chess	45%	90%
Coffee	60%	15%
Cooking	45%	90%
Cs	0%	90%
Philosophy	45%	90%
Corel5k	15%	30%
Corel16k001	0%	60%
Corel16k002	0%	75%
Genbase	15%	30%
Langlog	15%	15%
Medical	0%	30%
Slashdot	45%	30%
Yeast	30%	90%
Arts	0%	90%
Business	0%	90%
Entertainment	0%	90%
Health	15%	90%
Recreation	0%	90%
Reference	0%	90%
Science	30%	90%
Social	30%	90%

TABLE II
Optimal Value Distribution of k on All the Data Sets

Data set	Optimal k value
Bibtex, Chess, Coffee, Cooking, Cs, Philosophy, Corel16k001, Arts, Entertainment, Health, Recreation, Science, Social	5
Langlog, Medical, Reference	10
Genbase, Slashdot, Business	15
Birds, Chemistry, Corel5k, Corel16k002, Yeast	≥20

II. Experiments Using BSVM as the Classifier

In this part, instead of using ML-KNN as the classifier, we adopt BSVM (with linear kernel) [1] to evaluate the feature subsets selected by GRROfast, GRRO, and their six rivals. Similarly, top-50 features are selected by each algorithm, and the average result with 50 groups of feature subsets is recorded to make a comparison. TABLE III and TABLE IV report the experimental results of all the methods regarding each evaluation metric, from which we can observe that on *Hamming loss*, *ranking loss*, *coverage*, and *average precision*, *macro-F1*, and *micro-F1*, GRRO achieves better or comparable performance than the selected comparing methods on 23, 17, 17, 18, 20, and 16 out of 24 data sets respectively, while GRROfast compares favorably on 21, 17, 16, 18, 20, and 19 out of 24 data sets respectively. Obviously, GRRO or



(a) Bibtex (b) Birds (c) Chemistry (d) Chess
(e) Coffee (f) Cooking (g) Cs (h) Philosophy
(i) Corel5k (j) Corel16k001 (k) Corel16k002 (l) Genbase
(m) Langlog (n) Medical (o) Slashdot (p) Yeast
(q) Arts (r) Business (s) Entertainment (t) Health
(u) Recreation (v) Reference (w) Science (x) Social

Fig. 1. Parameter analysis of GRROfast on all the data sets.

TABLE III

COMPARISON RESULTS OF EIGHT MLFS METHODS (MEAN±STD. DEVIATION) IN TERMS OF *Hamming Loss*, *Ranking Loss*, AND *Coverage*

Data set	*Hamming Loss*↓							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	**0.0135±0.0004**	**0.0135±0.0005**	0.0150±0.0001	0.0140±0.0001	0.0146±0.0003	0.0137±0.0003	0.0137±0.0003	0.0137±0.0003
Birds	**0.0519±0.0025**	**0.0517±0.0046**	0.0692±0.0021	0.0590±0.0024	0.0613±0.0037	0.0551±0.0011	0.0520±0.0022	0.0524±0.0022
Chemistry	**0.0121±0.0001**	**0.0121±0.0005**	0.0122±0.0001	0.0123±0.0007	0.0122±0.0007	**0.0121±0.0001**	0.0122±0.0007	0.0122±0.0007
Chess	0.0100±0.0001	**0.0099±0.0001**	0.0102±0.0001	0.0102±0.0001	0.0102±0.0001	**0.0099±0.0001**	0.0100±0.0001	0.0100±0.0001
Coffee	**0.0153±0.0003**	**0.0157±0.0002**	0.0160±0.0002	0.0160±0.0001	0.0159±0.0002	0.0160±0.0002	0.0159±0.0002	0.0159±0.0002
Cooking	**0.0052±0.0001**	**0.0052±0.0001**	0.0055±0.0001	0.0055±0.0005	0.0055±0.0001	0.0053±0.0001	**0.0052±0.0001**	**0.0052±0.0001**
Cs	**0.0088±0.0001**	**0.0088±0.0001**	0.0091±0.0000	0.0089±0.0000	0.0090±0.0000	**0.0088±0.0001**	**0.0088±0.0001**	**0.0088±0.0001**
Philosophy	**0.0090±0.0002**	**0.0090±0.0001**	0.0094±0.0001	0.0092±0.0000	0.0094±0.0001	**0.0090±0.0001**	**0.0090±0.0001**	**0.0090±0.0001**
Corel5k	**0.0094±0.0000**	**0.0094±0.0000**	**0.0094±0.0000**	**0.0094±0.0000**	**0.0094±0.0000**	**0.0094±0.0000**	0.0095±0.0001	0.0095±0.0001
Corel16k001	**0.0187±0.0000**	**0.0187±0.0000**	**0.0187±0.0000**	0.0189±0.0008	**0.0187±0.0000**	0.0188±0.0000	0.0193±0.0017	0.0194±0.0019
Corel16k002	**0.0174±0.0000**	**0.0174±0.0000**	**0.0174±0.0000**	**0.0174±0.0000**	**0.0174±0.0000**	**0.0174±0.0000**	0.0178±0.0015	0.0175±0.0008
Genbase	**0.0051±0.0084**	**0.0053±0.0083**	0.0082±0.0080	0.0058±0.0081	0.0064±0.0089	0.0055±0.0082	0.0057±0.0083	0.0056±0.0082
Langlog	**0.0155±0.0001**	**0.0154±0.0001**	0.0157±0.0000	**0.0155±0.0000**	0.0157±0.0000	**0.0155±0.0001**	**0.0155±0.0001**	0.0156±0.0001
Medical	**0.0126±0.0029**	**0.0126±0.0029**	0.0182±0.0009	0.0144±0.0023	0.0144±0.0031	0.0140±0.0023	0.0144±0.0014	0.0140±0.0027
Slashdot	**0.0474±0.0016**	**0.0474±0.0016**	0.0496±0.0006	0.0488±0.0007	0.0481±0.0012	0.0476±0.0013	0.0475±0.0014	0.0475±0.0014
Yeast	0.2159±0.0068	**0.2133±0.0084**	0.2213±0.0094	0.2276±0.0144	0.2202±0.0056	**0.2133±0.0084**	0.2169±0.0069	0.2158±0.0071
Arts	**0.0572±0.0018**	**0.0572±0.0018**	0.0602±0.0011	0.0588±0.0008	0.0599±0.0018	0.0578±0.0015	0.0576±0.0018	0.0574±0.0018
Business	**0.0273±0.0004**	**0.0273±0.0005**	0.0284±0.0002	0.0279±0.0002	0.0281±0.0004	0.0275±0.0003	**0.0273±0.0005**	**0.0273±0.0005**
Entertainment	**0.0581±0.0026**	**0.0581±0.0026**	0.0654±0.0008	0.0610±0.0007	0.0647±0.0019	**0.0581±0.0022**	0.0593±0.0031	0.0594±0.0022
Health	**0.0395±0.0024**	**0.0397±0.0021**	0.0428±0.0017	0.0414±0.0017	0.0414±0.0022	0.0403±0.0026	0.0403±0.0022	0.0402±0.0023
Recreation	**0.0571±0.0015**	**0.0571±0.0016**	0.0649±0.0001	0.0581±0.0011	0.0615±0.0016	**0.0571±0.0015**	0.0572±0.0016	0.0572±0.0015
Reference	**0.0283±0.0010**	**0.0281±0.0012**	0.0295±0.0008	0.0301±0.0040	0.0291±0.0012	0.0284±0.0012	0.0284±0.0012	0.0284±0.0013
Science	**0.0334±0.0006**	**0.0333±0.0007**	0.0353±0.0003	0.0342±0.0003	0.0343±0.0003	0.0337±0.0005	0.0336±0.0005	0.0335±0.0006
Social	0.0240±0.0018	0.0239±0.0017	0.0250±0.0012	0.0254±0.0042	0.0243±0.0016	**0.0237±0.0017**	0.0244±0.0022	0.0241±0.0019
W/T/L (GRROfast)	/	/	21/3/0	21/3/0	21/3/0	13/8/3	18/6/0	18/5/1
W/T/L (GRRO)	/	/	20/4/0	22/2/0	21/3/0	14/9/1	20/4/0	20/4/0

Data set	*Ranking Loss*↓							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	0.2750±0.0334	0.2769±0.0297	0.3002±0.0172	0.2522±0.0454	0.2764±0.0283	**0.2502±0.0509**	0.2820±0.0359	0.2808±0.0378
Birds	**0.1427±0.0167**	**0.1423±0.0147**	0.1602±0.0092	0.1544±0.0079	0.1532±0.0131	0.1464±0.0155	0.1481±0.0170	0.1429±0.0184
Chemistry	0.1650±0.0084	0.1643±0.0089	0.1700±0.0059	0.1684±0.0060	0.1667±0.0072	**0.1638±0.0098**	0.1650±0.0090	0.1650±0.0090
Chess	0.1529±0.0096	0.1523±0.0096	0.1564±0.0072	0.1563±0.0073	0.1552±0.0081	**0.1511±0.0111**	0.1532±0.0097	0.1530±0.0095
Coffee	**0.1676±0.0245**	**0.1770±0.0199**	0.2030±0.0132	0.2035±0.0144	0.1979±0.0101	0.1970±0.0133	0.1903±0.0154	0.1909±0.0153
Cooking	**0.1953±0.0169**	0.1962±0.0155	0.2128±0.0076	0.2061±0.0075	0.2090±0.0098	0.1955±0.0169	0.1955±0.0171	**0.1954±0.0172**
Cs	0.1639±0.0083	0.1634±0.0095	0.1712±0.0050	0.1658±0.0079	0.1691±0.0066	**0.1617±0.0114**	0.1648±0.0088	0.1646±0.0090
Philosophy	**0.1418±0.0088**	**0.1416±0.0084**	0.1542±0.0022	0.1513±0.0029	0.1536±0.0026	0.1441±0.0078	0.1441±0.0080	0.1443±0.0078
Corel5k	**0.1421±0.0014**	**0.1418±0.0017**	0.1428±0.0012	0.1427±0.0012	0.1427±0.0012	0.1424±0.0015	0.1422±0.0015	0.1422±0.0016
Corel16k001	0.1937±0.0010	0.1937±0.0010	0.1927±0.0010	**0.1924±0.0015**	0.1926±0.0015	0.1946±0.0023	0.1943±0.0023	0.1941±0.0019
Corel16k002	0.1904±0.0014	0.1904±0.0013	0.1908±0.0009	**0.1897±0.0011**	0.1909±0.0012	0.1911±0.0013	0.1910±0.0024	0.1908±0.0020
Genbase	**0.0135±0.0218**	**0.0135±0.0219**	0.0154±0.0217	0.0140±0.0222	0.0155±0.0240	0.0136±0.0224	0.0149±0.0223	0.0145±0.0220
Langlog	**0.1647±0.0076**	**0.1624±0.0082**	0.1750±0.0034	0.1733±0.0043	0.1731±0.0049	0.1660±0.0094	0.1807±0.0044	0.1665±0.0093
Medical	**0.0576±0.0221**	**0.0576±0.0221**	0.0800±0.0154	0.0667±0.0173	0.0705±0.0262	0.0616±0.0233	0.0670±0.0263	0.0681±0.0251
Slashdot	**0.1750±0.0094**	**0.1750±0.0085**	0.1829±0.0059	0.1797±0.0065	0.1762±0.0105	0.1751±0.0100	0.1753±0.0098	0.1751±0.0098
Yeast	**0.1857±0.0076**	**0.1848±0.0074**	0.1967±0.0081	0.1996±0.0111	0.1925±0.0084	0.1859±0.0062	0.1869±0.0068	0.1865±0.0070
Arts	**0.1520±0.0101**	**0.1520±0.0100**	0.1600±0.0063	0.1585±0.0059	0.1569±0.0086	**0.1520±0.102**	0.1527±0.0096	0.1524±0.0092
Business	**0.0433±0.0022**	**0.0432±0.0020**	0.0478±0.0013	0.0459±0.0013	0.0468±0.0015	0.0447±0.0014	**0.0433±0.0022**	0.0436±0.0020
Entertainment	**0.1220±0.0092**	**0.1214±0.0102**	0.1376±0.0069	0.1344±0.0022	0.1403±0.0078	0.1227±0.0083	0.1270±0.0085	0.1290±0.0074
Health	**0.0615±0.0069**	**0.0611±0.0068**	0.0688±0.0042	0.0679±0.0039	0.0654±0.0056	0.0629±0.0072	0.0635±0.0062	0.0630±0.0066
Recreation	0.1769±0.0112	**0.1757±0.0119**	0.2168±0.0032	0.1831±0.0078	0.1978±0.0123	**0.1758±0.0107**	0.1772±0.0110	0.1769±0.0105
Reference	**0.0935±0.0083**	**0.0932±0.0078**	0.1008±0.0052	0.1014±0.0056	0.0982±0.0072	0.0938±0.0080	0.0952±0.0083	0.0940±0.0081
Science	**0.1393±0.0077**	**0.1390±0.0075**	0.1493±0.0038	0.1428±0.0049	0.1432±0.0059	0.1420±0.0051	0.1415±0.0052	0.1415±0.0061
Social	**0.0669±0.0056**	**0.0669±0.0052**	0.0709±0.0023	0.0709±0.0047	0.0709±0.0031	0.0691±0.0041	0.0710±0.0039	0.0705±0.0033
W/T/L (GRROfast)	/	/	24/0/0	21/0/3	23/0/1	18/1/5	22/2/0	22/2/0
W/T/L (GRRO)	/	/	23/0/1	22/0/2	22/0/2	18/1/5	23/0/1	23/0/1

Data set	*Coverage*↓							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	0.4221±0.0411	0.4226±0.0356	0.4469±0.0234	0.3779±0.0684	0.4119±0.0408	**0.3772±0.0738**	0.4252±0.0498	0.4232±0.0529
Birds	**0.2021±0.0158**	**0.2021±0.0156**	0.2214±0.0089	0.2187±0.0088	0.2099±0.0157	0.2101±0.0149	0.2099±0.0198	0.2026±0.0217
Chemistry	0.2658±0.0096	0.2653±0.0101	0.2709±0.0072	0.2692±0.0071	0.2668±0.0089	**0.2643±0.0117**	0.2659±0.0106	0.2660±0.0105
Chess	0.2816±0.0112	0.2809±0.0100	0.2847±0.0090	0.2847±0.0091	0.2835±0.0097	**0.2786±0.0137**	0.2820±0.0116	0.2817±0.0110
Coffee	**0.2655±0.0133**	**0.2817±0.0140**	0.3019±0.0114	0.3055±0.0154	0.3043±0.0133	0.3072±0.0154	0.3033±0.0129	0.3046±0.0135
Cooking	**0.3226±0.0200**	**0.3226±0.0173**	0.3430±0.0092	0.3362±0.0080	0.3382±0.0118	**0.3226±0.0208**	0.3228±0.0201	**0.3226±0.0204**
Cs	0.2941±0.0103	0.2941±0.0118	0.3000±0.0072	0.2941±0.0115	0.2973±0.0096	**0.2899±0.0159**	0.2947±0.0113	0.2947±0.0121
Philosophy	**0.2513±0.0104**	**0.2510±0.0091**	0.2661±0.0024	0.2622±0.0027	0.2652±0.0026	0.2532±0.0091	0.2532±0.0094	0.2536±0.0090
Corel5k	**0.3216±0.0011**	**0.3215±0.0014**	0.3245±0.0026	0.3244±0.0025	0.3243±0.0025	0.3225±0.0026	0.3223±0.0014	0.3223±0.0015
Corel16k001	0.3673±0.0035	0.3661±0.0025	0.3656±0.0014	0.3657±0.0013	**0.3654±0.0021**	0.3701±0.0046	0.3691±0.0038	0.3684±0.0026
Corel16k002	0.3642±0.0021	0.3644±0.0034	0.3647±0.0014	**0.3630±0.0013**	0.3645±0.0018	0.3654±0.0022	0.3660±0.0044	0.3658±0.0040
Genbase	**0.0281±0.0231**	**0.0279±0.0233**	0.0312±0.0224	0.0289±0.0238	0.0310±0.0254	0.0283±0.0241	0.0298±0.0238	0.0294±0.0234
Langlog	**0.1701±0.0053**	**0.1675±0.0056**	0.1795±0.0022	0.1779±0.0024	0.1785±0.0035	0.1714±0.0070	0.1827±0.0025	0.1727±0.0070
Medical	**0.0822±0.0231**	**0.0822±0.0231**	0.1061±0.0159	0.0914±0.0180	0.0956±0.0277	0.0864±0.0246	0.0928±0.0245	0.0930±0.0263
Slashdot	**0.1916±0.0083**	**0.1915±0.0075**	0.1984±0.0052	0.1959±0.0056	0.1926±0.0096	0.1917±0.0089	0.1919±0.0087	0.1917±0.0088
Yeast	**0.4760±0.0068**	**0.4751±0.0065**	0.4946±0.0182	0.4924±0.0166	0.4867±0.0098	0.4762±0.0048	0.4786±0.0081	0.4804±0.0089
Arts	0.2126±0.0088	**0.2125±0.0089**	0.2202±0.0057	0.2188±0.0050	0.2173±0.0080	**0.2125±0.0090**	0.2130±0.0084	0.2130±0.0080
Business	**0.0806±0.0023**	**0.0806±0.0023**	0.0859±0.0014	0.0841±0.0017	0.0845±0.0013	0.0822±0.0016	0.0809±0.0023	0.0811±0.0021
Entertainment	**0.1600±0.0078**	**0.1584±0.0094**	0.1727±0.0071	0.1716±0.0024	0.1760±0.0076	0.1607±0.0075	0.1649±0.0069	0.1661±0.0067
Health	**0.1068±0.0076**	**0.1059±0.0081**	0.1145±0.0048	0.1141±0.0042	0.1112±0.0061	0.1078±0.0083	0.1094±0.0068	0.1088±0.0074
Recreation	0.2212±0.0101	0.2195±0.0102	0.2547±0.0036	0.2265±0.0068	0.2388±0.0118	**0.2192±0.0093**	0.2209±0.0093	0.2210±0.0092
Reference	**0.1100±0.0076**	**0.1092±0.0076**	0.1166±0.0054	0.1171±0.0057	0.1141±0.0073	0.1101±0.0077	0.1113±0.0082	0.1101±0.0080
Science	**0.1760±0.0081**	**0.1760±0.0076**	0.1858±0.0036	0.1802±0.0047	0.1806±0.0052	0.1796±0.0046	0.1788±0.0052	0.1789±0.0059
Social	**0.0922±0.0065**	**0.0925±0.0059**	0.0965±0.0026	0.0966±0.0052	0.0965±0.0031	0.0952±0.0042	0.0967±0.0043	0.0961±0.0035
W/T/L (GRROfast)	/	/	23/0/1	21/1/2	22/0/2	17/1/6	23/0/1	22/1/1
W/T/L (GRRO)	/	/	23/0/1	20/1/3	22/0/2	17/2/5	24/0/0	23/1/0

TABLE IV

COMPARISON RESULTS OF EIGHT MLFS METHODS (MEAN±STD. DEVIATION) IN TERMS OF *Average Precision*, *Macro-F1*, AND *Micro-F1*

Data set	*Average Precision*↑							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	**0.2771±0.0599**	0.2626±0.0557	0.1613±0.0188	0.2443±0.0349	0.1959±0.0309	**0.2718±0.0611**	0.2447±0.0469	0.2455±0.0479
Birds	**0.6812±0.0293**	**0.6817±0.0322**	0.6208±0.0106	0.6596±0.0107	0.6306±0.0260	0.6810±0.0249	0.6736±0.0323	0.6792±0.0288
Chemistry	**0.2705±0.0235**	**0.2684±0.0240**	0.2466±0.0093	0.2477±0.0155	0.2497±0.0144	0.2682±0.0205	0.2648±0.0236	0.2650±0.0233
Chess	**0.3120±0.0380**	**0.3202±0.0415**	0.2553±0.0183	0.2569±0.0197	0.2623±0.0234	0.3099±0.0358	0.3057±0.0396	0.3067±0.0384
Coffee	**0.4097±0.0688**	**0.3908±0.0631**	0.3021±0.0344	0.2993±0.0344	0.3112±0.0366	0.3269±0.0323	0.3538±0.0414	0.3527±0.0413
Cooking	**0.2395±0.0632**	**0.2579±0.0600**	0.1306±0.0190	0.1764±0.0208	0.1470±0.0271	0.2356±0.0595	0.2368±0.0633	0.2371±0.0630
Cs	**0.2876±0.0299**	**0.2883±0.0368**	0.2072±0.0044	0.2556±0.0144	0.2213±0.0132	0.2807±0.0304	0.2767±0.0287	0.2777±0.0275
Philosophy	**0.3398±0.0488**	**0.3421±0.0399**	0.2453±0.0061	0.2679±0.0068	0.2465±0.0075	0.3292±0.0351	0.3312±0.0357	0.3311±0.0357
Corel5k	**0.2223±0.0043**	**0.2244±0.0065**	0.2156±0.0016	0.2158±0.0014	0.2163±0.0016	0.2184±0.0024	0.2170±0.0096	0.2154±0.0136
Corel16k001	0.2485±0.0010	0.2485±0.0010	0.2487±0.0013	**0.2497±0.0092**	0.2485±0.0016	0.2461±0.0089	0.2425±0.0187	0.2412±0.0202
Corel16k002	0.2426±0.0031	0.2408±0.0023	0.2410±0.0019	**0.2436±0.0027**	0.2408±0.0015	0.2417±0.0014	0.2369±0.0072	0.2404±0.0093
Genbase	**0.9552±0.0964**	0.9514±0.0967	0.9488±0.0979	**0.9524±0.0981**	0.9492±0.1047	0.9522±0.0980	0.9455±0.1003	0.9478±0.0980
Langlog	**0.2940±0.0178**	**0.2987±0.0288**	0.2437±0.0083	0.2543±0.0147	0.2437±0.0077	0.2934±0.0342	0.2430±0.0158	0.2905±0.0271
Medical	**0.7964±0.0907**	**0.7964±0.0907**	0.6791±0.0482	0.7531±0.0716	0.7441±0.1033	0.7665±0.0871	0.7575±0.0888	0.7583±0.0937
Slashdot	**0.4819±0.0325**	0.4802±0.0292	0.4453±0.0162	0.4636±0.0193	0.4742±0.0313	**0.4805±0.0309**	0.4793±0.0299	0.4795±0.0299
Yeast	0.7393±0.0116	**0.7412±0.0105**	0.7270±0.0111	0.7169±0.0240	0.7320±0.0106	0.7402±0.0092	0.7396±0.0102	**0.7404±0.0096**
Arts	**0.5269±0.0336**	**0.5267±0.0324**	0.4959±0.0251	0.5116±0.0222	0.5053±0.0322	0.5260±0.0326	0.5222±0.0314	0.5250±0.0305
Business	0.8724±0.0036	0.8725±0.0036	0.8639±0.0028	0.8698±0.0027	0.8671±0.0032	0.8707±0.0024	**0.8727±0.0037**	0.8725±0.0035
Entertainment	0.5781±0.0392	**0.5825±0.0419**	0.5173±0.0171	0.5286±0.0088	0.5123±0.0186	**0.5795±0.0334**	0.5600±0.0378	0.5551±0.0324
Health	**0.7147±0.0251**	**0.7126±0.0241**	0.6691±0.0135	0.6861±0.0134	0.6925±0.0229	0.7076±0.0264	0.7043±0.0234	0.7061±0.0246
Recreation	**0.5276±0.0340**	**0.5278±0.0364**	0.3876±0.0066	0.5038±0.0224	0.4510±0.0325	0.5273±0.0316	0.5256±0.0346	0.5267±0.0329
Reference	0.6030±0.0234	**0.6142±0.0313**	0.5871±0.0101	0.5834±0.0255	0.5945±0.0207	0.6126±0.0236	0.6084±0.0255	**0.6139±0.0233**
Science	**0.4775±0.0247**	**0.4801±0.0241**	0.4224±0.0182	0.4594±0.0132	0.4586±0.0161	0.4658±0.0186	0.4693±0.0204	0.4714±0.0225
Social	**0.7180±0.0200**	**0.7183±0.0193**	0.7005±0.0097	0.6964±0.0411	0.7079±0.0129	0.7119±0.0189	0.7073±0.0169	0.7097±0.0151
W/T/L (GRROfast)	/	/	23/0/1	22/0/2	23/1/0	21/0/3	21/0/3	21/0/3
W/T/L (GRRO)	/	/	23/0/1	21/0/3	22/2/0	20/0/4	23/0/1	23/1/0

Data set	*Macro-F1*↑							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	**0.0492±0.0270**	**0.0483±0.0312**	0.0014±0.0016	0.0195±0.0050	0.0050±0.0030	0.0327±0.0184	0.0314±0.0179	0.0317±0.0183
Birds	**0.1309±0.0442**	**0.1471±0.0583**	0.0630±0.0345	0.1269±0.0303	0.0792±0.0413	0.1192±0.0292	0.1250±0.0549	0.1297±0.0610
Chemistry	**0.0098±0.0070**	**0.0092±0.0064**	0.0016±0.0011	0.0025±0.0009	0.0023±0.0015	0.0065±0.0037	0.0080±0.0055	0.0079±0.0054
Chess	0.0159±0.0066	**0.0210±0.0088**	0.0047±0.0031	0.0053±0.0028	0.0059±0.0044	0.0155±0.0069	0.0157±0.0066	**0.0160±0.0066**
Coffee	**0.5365±0.0191**	**0.5244±0.0121**	0.4895±0.0027	0.4891±0.0021	0.4933±0.0073	0.4937±0.0054	0.5053±0.0106	0.5057±0.0109
Cooking	**0.0240±0.0148**	**0.0288±0.0149**	0.0025±0.0022	0.0091±0.0024	0.0044±0.0034	0.0222±0.0124	0.0238±0.0148	0.0239±0.0148
Cs	**0.0203±0.0079**	**0.0240±0.0109**	0.0015±0.0014	0.0089±0.0021	0.0021±0.0016	0.0156±0.0065	0.0164±0.0075	0.0167±0.0073
Philosophy	**0.0289±0.0125**	**0.0306±0.0126**	0.0019±0.0007	0.0069±0.0011	0.0019±0.0007	0.0218±0.0084	0.0220±0.0090	0.0219±0.0087
Corel5k	**0.2997±0.0018**	**0.3004±0.0026**	0.2968±0.0000	0.2967±0.0004	0.2968±0.0000	0.2970±0.0004	0.2972±0.0005	0.2972±0.0004
Corel16k001	**0.0022±0.0005**	**0.0013±0.0002**	0.0000±0.0000	0.0000±0.0002	0.0000±0.0004	0.0003±0.0002	0.0006±0.0006	0.0005±0.0006
Corel16k002	**0.0043±0.0022**	0.0033±0.0015	0.0000±0.0000	**0.0036±0.0011**	0.0000±0.0000	0.0034±0.0014	0.0032±0.0012	0.0030±0.0013
Genbase	**0.6671±0.1770**	**0.6707±0.1787**	0.5931±0.1476	0.6241±0.1556	0.6320±0.1801	0.6608±0.1776	0.6639±0.1831	0.6669±0.1812
Langlog	**0.1192±0.0011**	**0.1201±0.0046**	0.1067±0.0000	0.1136±0.0010	0.1067±0.0000	0.1189±0.0023	0.1116±0.0017	0.1187±0.0029
Medical	**0.4002±0.0829**	**0.4011±0.0829**	0.2607±0.0208	0.3128±0.0363	0.3348±0.0800	0.3570±0.0658	0.3429±0.0705	0.3465±0.0786
Slashdot	**0.2408±0.0359**	**0.2420±0.0370**	0.1954±0.0153	0.2132±0.0204	0.2234±0.0275	0.2366±0.0338	0.2357±0.0340	0.2357±0.0332
Yeast	0.2922±0.0507	0.2934±0.0484	0.2661±0.0508	0.2536±0.0506	0.2758±0.0553	0.2941±0.0506	0.2916±0.0498	**0.2973±0.0424**
Arts	**0.0800±0.0255**	**0.0806±0.0248**	0.0515±0.0164	0.0635±0.0126	0.0573±0.0242	0.0759±0.0224	0.0766±0.0233	0.0793±0.0239
Business	**0.1645±0.0166**	0.1612±0.0192	0.1146±0.0082	0.1438±0.0081	0.1416±0.0213	0.1543±0.0144	0.1617±0.0209	**0.1633±0.0190**
Entertainment	0.1048±0.0274	**0.1071±0.0298**	0.0466±0.0151	0.0735±0.0072	0.0463±0.0232	**0.1056±0.023**3	0.0874±0.0273	0.0873±0.0211
Health	**0.2019±0.0301**	**0.2050±0.0228**	0.1678±0.0280	0.1853±0.0300	0.1857±0.0309	0.1903±0.0298	0.1990±0.0332	0.1981±0.0339
Recreation	0.1108±0.0236	0.1118±0.0235	0.0021±0.0032	0.1024±0.0169	0.0545±0.0261	**0.1143±0.0216**	0.1113±0.0238	0.1122±0.0225
Reference	**0.1201±0.0166**	**0.1240±0.0166**	0.0921±0.0044	0.0926±0.0037	0.0996±0.0150	0.1180±0.0173	0.1136±0.0190	0.1194±0.0174
Science	**0.0483±0.0182**	**0.0480±0.0175**	0.0146±0.0075	0.0395±0.0127	0.0320±0.0079	0.0370±0.0124	0.0395±0.0131	0.0408±0.0142
Social	**0.1237±0.0257**	**0.1330±0.0309**	0.1026±0.0097	0.1057±0.0087	0.0973±0.0089	0.1121±0.0136	0.1011±0.0137	0.1029±0.0114
W/T/L (GRROfast)	/	/	24/0/0	24/0/0	24/0/0	21/0/3	23/0/1	21/0/3
W/T/L (GRRO)	/	/	24/0/0	23/0/1	24/0/0	21/0/3	23/0/1	21/0/3

Data set	*Micro-F1*↑							
	GRROfast	GRRO	PMU	MDMR	FIMF	SCLS	MICO	LRFS
Bibtex	**0.2301±0.0660**	**0.2244±0.0754**	0.0163±0.0187	0.1629±0.0153	0.0776±0.0373	0.1914±0.0449	0.1911±0.0469	0.1915±0.0481
Birds	**0.5152±0.0387**	**0.5162±0.1252**	0.2385±0.1385	0.4849±0.0156	0.4075±0.0849	0.4880±0.0220	0.5070±0.0416	0.5109±0.0504
Chemistry	**0.0509±0.0282**	**0.0440±0.0224**	0.0152±0.0130	0.0257±0.0095	0.0207±0.0139	0.0421±0.0222	0.0414±0.0212	0.0414±0.0214
Chess	0.1567±0.0393	**0.1695±0.0416**	0.0795±0.0357	0.0896±0.0190	0.0832±0.0390	0.1558±0.0420	0.1554±0.0409	**0.1570±0.0397**
Coffee	**0.1585±0.0486**	0.1381±0.0336	0.0091±0.0148	0.0080±0.0131	0.0832±0.0328	0.1558±0.0192	0.1554±0.0433	**0.1570±0.0450**
Cooking	**0.1571±0.0679**	**0.1788±0.0602**	0.0173±0.0174	0.0835±0.0180	0.0376±0.0281	0.1511±0.0597	0.1547±0.0675	0.1549±0.0668
Cs	**0.1222±0.0363**	**0.1258±0.0432**	0.0106±0.0063	0.0671±0.0151	0.0189±0.0110	0.1065±0.0355	0.1051±0.0393	0.1063±0.0374
Philosophy	**0.1662±0.0529**	**0.1722±0.0435**	0.0407±0.0146	0.0900±0.0097	0.0407±0.0146	0.1622±0.0385	0.1660±0.0404	0.1653±0.0399
Corel5k	**0.0137±0.0102**	**0.0154±0.0128**	0.0000±0.0000	0.0000±0.0000	0.0000±0.0000	0.0005±0.0013	0.0014±0.0016	0.0014±0.0016
Corel16k001	**0.0042±0.0018**	0.0040±0.0018	0.0000±0.0000	0.0003±0.0018	0.0000±0.0002	0.0021±0.0021	0.0041±0.0038	**0.0042±0.0041**
Corel16k002	**0.0075±0.0028**	0.0072±0.0030	0.0000±0.0000	0.0041±0.0008	0.0000±0.0000	0.0071±0.0031	**0.0075±0.0024**	0.0071±0.0027
Genbase	**0.9274±0.1394**	**0.9262±0.1352**	0.8933±0.1331	0.9202±0.1337	0.9114±0.1470	0.9238±0.1355	0.9199±0.1357	0.9224±0.1338
Langlog	**0.1131±0.0178**	**0.1154±0.0189**	0.0000±0.0000	0.0890±0.0129	0.0000±0.0000	0.1114±0.0174	0.0463±0.0156	0.1031±0.0161
Medical	**0.7387±0.0905**	**0.7417±0.0905**	0.5707±0.0389	0.6910±0.0799	0.6907±0.1046	0.7045±0.0772	0.7019±0.0836	0.7050±0.0886
Slashdot	**0.2636±0.0613**	**0.2637±0.0621**	0.1735±0.0226	0.2098±0.0289	0.2339±0.0459	0.2622±0.0588	0.2603±0.0574	0.2595±0.0561
Yeast	0.6080±0.0408	0.6083±0.0375	0.5821±0.0348	0.5693±0.0299	0.6023±0.0408	0.6018±0.0357	0.6079±0.0402	**0.6151±0.0341**
Arts	**0.2190±0.0632**	**0.2189±0.0626**	0.1390±0.0453	0.1784±0.0351	0.1472±0.0685	0.2089±0.0624	0.2085±0.0630	0.2157±0.0632
Business	**0.6933±0.0076**	0.6924±0.0079	0.6749±0.0033	0.6834±0.0039	0.6809±0.0067	0.6917±0.0069	0.6920±0.0092	**0.6931±0.0085**
Entertainment	0.2873±0.0769	0.2873±0.0802	0.0975±0.0273	0.2116±0.0273	0.1128±0.0561	**0.2920±0.0647**	0.2505±0.0818	0.2568±0.0676
Health	**0.5241±0.0270**	**0.5243±0.0242**	0.4328±0.0219	0.4726±0.0204	0.4930±0.0255	0.5017±0.0248	0.5158±0.0268	0.5157±0.0281
Recreation	0.2476±0.0492	0.2477±0.0501	0.0052±0.0080	0.2198±0.0341	0.1158±0.0542	**0.2500±0.0464**	0.2462±0.0515	0.2477±0.0483
Reference	**0.3851±0.0359**	**0.3900±0.0437**	0.3365±0.0325	0.3361±0.0338	0.3458±0.0479	0.3787±0.0456	0.3769±0.0472	0.3830±0.0485
Science	**0.1659±0.0435**	**0.1676±0.0492**	0.0381±0.0218	0.1166±0.0285	0.1047±0.0217	0.1444±0.0383	0.1503±0.0390	0.1532±0.0430
Social	0.4805±0.0815	0.4817±0.0798	0.4398±0.0623	0.4481±0.0659	0.4613±0.0745	**0.4843±0.0650**	0.4589±0.0830	0.4707±0.0757
W/T/L (GRROfast)	/	/	24/0/0	24/0/0	24/0/0	21/0/3	23/1/0	20/1/3
W/T/L (GRRO)	/	/	24/0/0	24/0/0	24/0/0	20/0/4	21/0/3	19/1/4

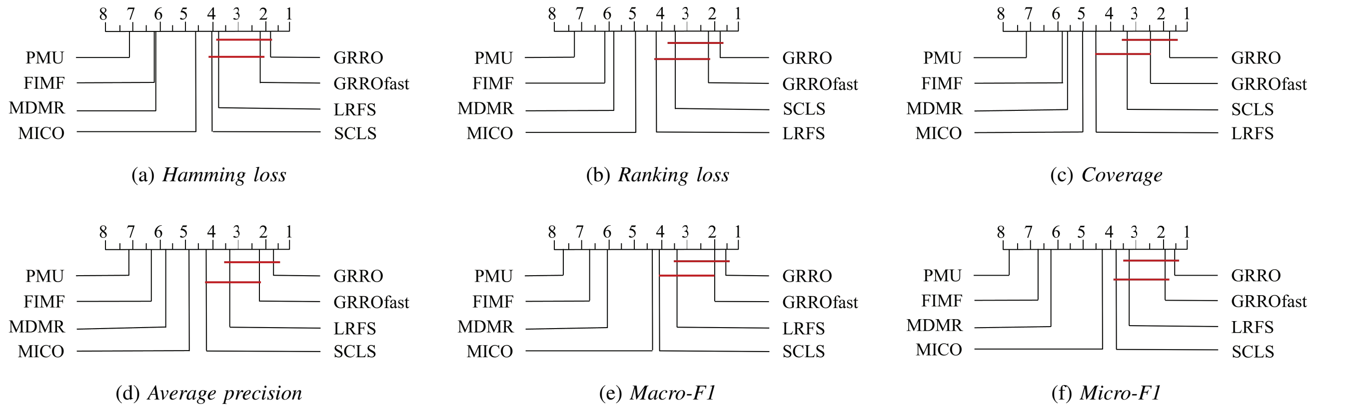


Fig. 2. Comparison of GRRO or GRROfast (control method) against the selected comparing methods with the Nemenyi test ($CD = 2.1432$ at 0.05 significance level).

TABLE V
FRIEDMAN STATISTICS F_F AND THE CRITICAL VALUE ON EVALUATION METRICS (# COMPARING ALGORITHMS $c = 8$, # DATA SETS $N = 24$)

Evaluation metric	F_F	Critical value($\alpha = 0.05$)
Hamming loss	36.5926	
Ranking loss	36.9489	
Coverage	27.3658	≈2.01
Average precision	37.1889	
Macro-F1	90.5659	
Micro-F1	124.6688	

GRROfast achieves the optimal average rank on each metric. We further compare the two proposed algorithms GRRO and GRROfast. GRRO is superior to GRROfast on most of data sets. Specially, on all the 24 data sets, GRRO outperforms GRROfast on 17 and 15 data sets in terms of *macro-F1* and *micro-F1* respectively, which also has the advantages on the other metrics. This suggests that GRROfast reduces computational cost at the expense of learning performance. Finally, we can observe from the W/T/L record that on *Hamming loss*, *ranking loss*, *coverage*, and *average precision*, *macro-F1*, and *micro-F1*, the selected comparing methods obtain better performance than the proposed algorithms on up to 3, 5, 5, 3, 3, and 3 out of 24 data sets, respectively. To summarize, the proposed algorithms are proved to be effective for multi-label feature selection, and compare better than some other well-established methods.

Moreover, we use Friedman test [2], [3] to validate whether GRROfast, GRRO, and their rivals have no significant difference, and the Friedman statistic F_F and the corresponding critical value on each evaluation metric are shown in TABLE V. According to TABLE V, the null hypothesis at the Friedman test is clearly rejected with respect to each metric at significance level $\alpha = 0.05$. Hence, we proceed with the Nemenyi test [2] as a post-hoc test and the CD diagram is shown in Fig. 2. We can see from Fig. 2 that GRRO achieves the lowest average rank on all evaluation metrics, and GRRO and GRROfast have no significant difference. In addition, GRRO achieves significantly better performance against all the selected comparing methods, while GRROfast is comparable to LRFS and SCLS, but significantly performs better than the remaining four comparing methods.

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